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Input Disturbance Estimation Using a General Structured Observer

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This paper presents the characteristics of a general structured observer and presents an estimation algorithm for a system with external disturbances which are added to the input of the system. By using a disturbance model, the general structured observer can estimate the states of the system in spite of disturbances, where the system is affected from external disturbances. Also, the general structured observer can include the function of a PI observer or high gain observer by properly adjusting the observer's gain matrices. The existence condition for the observer is derived, which can be checked by the system's observability condition and the pole-zero cancellation of the system's polynomial matrix. Through a numerical example, it is verified that the proposed observer is effective estimating the states of the system and the input disturbance.

Key Words : General Structured Observer, Input Disturbance, Observability

Nomenclature

$x(t)$	: State vector of system
$\hat{x}(t)$	: Estimated state vector of system
$d(t)$	: Input disturbance vector
$\hat{d}(t)$	: Estimated disturbance vector
$e_x(t)$	: Error vector between real states and estimated states
$\zeta(t)$	: Transformed estimated error vector
$\xi(t)$	: Transformed error vector
K, H, M, N	: Gain matrices of general structured observer

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1. Introduction

In recent years, the problem of estimating the states of uncertain dynamical systems subjected to external disturbances has been a topic of considerable interest. In the area of observer design, many researchers have assumed that the estimated state vector is available to construct the observer-based controller. In practice, it is not always possible to use the estimated state vector instead of real states, because real plants normally have disturbances. Therefore the state vector should be estimated by a proper observer scheme based on the system's uncertainties or disturbances.

The conventional Luenberger type observer only works for estimating the state vector of a linear time-invariant system. If there are system uncertainties or disturbances, it is necessary to design an adequate observer design method. Many kinds of observers, e.g., unknown-input observer, PI observer, adaptive observer and nonlinear observer, has been presented in the literature. The design of a disturbance decoupled bilinear observer for nonlinear systems have been studied by Yi (1995). For system uncertainties, an observer-based control algorithm is presented for a system with nonlinear uncertainties (Saberi, 1990), where uncertain elements in the plant are modeled as cone bounded non-linearities.

Also, the design method for a discrete observer has been proposed by Edwards and Spurgeon (1994), where the upper bounded value of uncertain dynamical systems is considered. Observer-based positive real control of uncertain linear systems is presented by Mahmoud et al. (1999), where norm-bounded uncertainties are considered. As an application of observer scheme for the real plants, a Luenberger type observer is designed for control of constrained mechanical systems by Hou et al. (1999). A general structured observer has been proposed for linear systems with unknown inputs by Chang (1997), which consists of a PI observer only as an internal model.

Recently, PI observers have received attention for estimating the states of a system with step or low frequency domain disturbances. Having the properties of disturbance cancellation, a PI observer has been applied to the design of robust control systems (Kim, 1996), and is utilized for the FDI design method for fault detection and isolation of actuator failures (Kim et al., 1997). For observer-based monitoring systems, a discrete well-conditioned state observer based on a unified main index was developed by Huh et al. (1997). The estimated states of the system by using a conventional observer for a real control system with external disturbances always have estimation errors between the real states and estimated states. Thus, it is necessary to design a general structured type of observer that is easily applicable to real plants affected by disturbances.

In this paper, we propose a general structured observer that can estimate not only the states of a system but also the input disturbances to the system with external disturbances. Also, the existence condition for the proposed observer is presented. The condition of a general structured observer is checked by the system's observability and the pole-zero cancellation of the system's polynomial matrix. Through a numerical example, we verify the effective characteristics for estimating the states of the system and the external disturbance.

2. General Structured Observer

Consider the following continuous time-invariant system described by

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1a)$$

$$y(t) = Cx(t) \quad (1b)$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$ are the state vector, the control inputs, and the measured outputs respectively. Matrices A, B, and C are of known with appropriate dimensions. It is assumed that (A, C) is observable.

Consider the following general structured observer represented by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K\zeta(t) + H\varepsilon(t) \quad (2a)$$

$$\dot{\zeta}(t) = N\zeta(t) + M\varepsilon(t) \quad (2b)$$

$$\varepsilon(t) = y(t) - C\hat{x}(t) \quad (2c)$$

where $\hat{x}(t) \in R^n$, $\zeta(t) \in R^r$, and $\varepsilon(t) \in R^p$ are the estimated state vector, the transformed vector, and the estimated error vector respectively. Matrices K, H, N, and M are general structured observer's gain.

The block diagram of general structured observer is given by Fig. 1.

Definition 1 : The system in Eq. (2) is said to be a general structured observer for the system in Eq. (1) if the following relations exist for any initial conditions $x(0)$, $\hat{x}(0)$, and for any input $u(\cdot)$.

$$\lim_{t \to \infty} \{x(t) - \hat{x}(t)\} = 0 \quad (3a)$$

$$\lim_{t \to \infty} \zeta(t) = 0 \quad (3b)$$

■

Eq. (3) shows that the estimation error converge on zero at $t \to \infty$.

Under the above definition, we have the following relationship between the system and the observer.

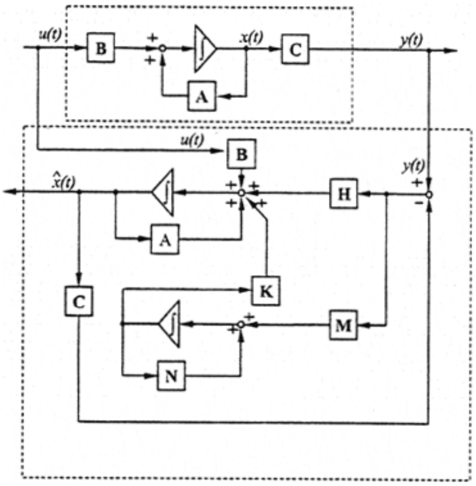

Fig. 1 Block diagram of general structured observer scheme

Theorem 1 : The system in Eq. (2) is said to be a general structured observer for the system in Eq. (1) if

$$\text{Re } \lambda_i \begin{bmatrix} A-HC & -K \\ MC & N \end{bmatrix} < 0,$$
$$\text{for all } i(i=1, 2, \cdots, n+r) \quad (4)$$

■

Proof : Define estimation error by

$$e_x(t) = x(t) - \hat{x}(t) \quad (5)$$

Using Eq. (1) and Eq. (2), we have

$$\dot{e}_x(t) = \dot{x}(t) - \dot{\hat{x}}(t) = (A-HC)\,e_x(t) - K\zeta(t) \quad (6)$$

Then, an augmented system is constructed by Eq. (2b) and (6) as follows:

$$\begin{bmatrix} \dot{e}_x(t) \\ \dot{\zeta}(t) \end{bmatrix} = \begin{bmatrix} A-HC & -K \\ MC & N \end{bmatrix} \begin{bmatrix} e_x(t) \\ \zeta(t) \end{bmatrix} \quad (7)$$

Thus, under the condition of Eq. (4), $e_x(t) \to 0$ and $\zeta(t) \to 0 (t \to \infty)$. So, this proof is completed. □

If the gain matrices of general structured observer are selected as $M=I$ and $N=0$, then the general structured observer can be transformed into the conventional PI observer (Kim, 1996) as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K\zeta(t) + H\varepsilon(t) \quad (8a)$$
$$\dot{\zeta}(t) = \varepsilon(t) \quad (8b)$$
$$\varepsilon(t) = y(t) - C\hat{x}(t) \quad (8c)$$

where the matrices K and H denote a proportional and a integral gains in PI observer respectively.

Also by selecting $K=0$, Eq. (8) will be changed into the conventional high gain observer. Accordingly, various observers can be formulated by adjusting the general structured observer's gains arbitrarily.

3. Estimation Algorithm for Input Disturbance Using a General Structured Observer

To estimate the input disturbance of system by using the general structured observer, let us consider the following linear continuous system with external disturbance at input of the plant.

$$\dot{x}(t) = Ax(t) + Bu(t) + Dd(t) \quad (9a)$$
$$y(t) = Cx(t) \quad (9b)$$
$$\dot{d}(t) = Wd(t) \quad (9c)$$

where, $d(t) \in R^r$ is external disturbance vector and W is system matrix of external disturbance.

We apply the general structured observer to the system described by Eq. (9). To estimate input disturbance by using general structured observer, we define the error function of estimated state as Eq. (5).

Eq. (10) can be expressed by differentiating Eq. (5), and Eq. (11) is given by system Eq. (2b).

$$\dot{e}_x(t) = \dot{x}(t) - \dot{\hat{x}}(t) = (A-HC)\,e_x(t) - K\zeta(t) + Dd(t) \quad (10)$$
$$\dot{\zeta}(t) = N\zeta(t) + MCe_x(t) \quad (11)$$

We design the gain matrix of general structured observer as follows:

$$K = D \quad (12)$$

and define the variable $\zeta(t)$ as

$$\dot{\xi}(t) = \dot{\zeta}(t) - \dot{d}(t) \quad (13)$$

Then using Eqs. (12) ~ (13), Eq. (10) is rearranged as

$$\dot{e}_x(t) = (A-HC)\,e_x(t) - K\xi(t) \quad (14)$$

And, Eq. (13) can be rewritten as

$$\begin{aligned}\dot{\xi}(t)&=\dot{\zeta}(t)-\dot{d}(t)\\&=N\zeta(t)+MCe_x(t)-Wd(t)\end{aligned} \tag{15}$$

Let us design the matrix N as

$$N=W \tag{16}$$

Then, Eq. (15) is rewritten as

$$\dot{\xi}(t)=MCe_x(t)+N\xi(t) \tag{17}$$

Thus, from Eqs. (14) and (17), the augmented system is obtained as next.

$$\begin{bmatrix}\dot{e}(t)\\ \dot{\xi}(t)\end{bmatrix}=\begin{bmatrix}A-HC & -K\\ MC & N\end{bmatrix}\begin{bmatrix}e_x(t)\\ \xi(t)\end{bmatrix} \tag{18}$$

If the observer's gain H and M are designed so that $\begin{bmatrix}A-HC & -K\\ MC & N\end{bmatrix}$ is stable, then $e_x(t)$, $\xi(t)$ $\rightarrow 0$ at $t\rightarrow\infty$ and the disturbance can be obtained as follows:

$$\hat{d}(t)=\zeta(t) \tag{19}$$

4. Existence Condition of General Structured Observer with Input Disturbance

The existence condition of general structured observer's gain K, H, N, and M which satisfies the Theorem 1 will be treated in this section. The augmented system matrix of Eq. (18) can be rewritten as follows:

$$\begin{bmatrix}A-HC & -K\\ MC & N\end{bmatrix}=\begin{bmatrix}A & -K\\ 0 & N\end{bmatrix}-\begin{bmatrix}H\\ -M\end{bmatrix}[C\ \ 0] \tag{20}$$

From the above equation, if the matrix $\left(\begin{bmatrix}A & -K\\ 0 & N\end{bmatrix}, [C\ \ 0]\right)$ is observable, then we can obtain the matrices H and M which satisfy the condition of Theorem 1. The stable gain matrices can be calculated by design of LQG or pole placement etc..

Let us define that the $\lambda_A(i)$ are eigenvalues of A, $i=1, 2, \cdots, n$ and $\lambda_N(j)$ is eigenvalues of N, $j=1, 2, \cdots, r$.

Theorem 2 : If the matrices A and N have distinct eigenvalues, $\left(\begin{bmatrix}A & -K\\ 0 & N\end{bmatrix}, [C\ \ 0]\right)$ is completely observable if and only if,

(i) (A, C) is observable.

(ii) the polynomial matrix $[-C\ \mathrm{adj}(sI-A)\ K\ \mathrm{adj}(sI-N)]$ do not have common factor of $(s-\lambda_N(j))$; $j=1, 2, \cdots, r$ i.e. there is no zero cancellation of the eigenvalues of N.

(iii) all eigenvalues of A and N are simple or, if repeated, then the repeated eigenvalue must have a simple degeneracy, $q=1$ associated with it, i.e. the following condition must be satisfied

$$\mathrm{rank}\left(\lambda I-\begin{bmatrix}A & -K\\ 0 & N\end{bmatrix}\right)=n+r-1 \text{ for all } \lambda \tag{21}$$

where $q=(n+r)-\mathrm{rank}\left(\lambda I-\begin{bmatrix}A & -K\\ 0 & N\end{bmatrix}\right)$ ■

Proof : The transfer matrix of the system is obtained as follows:

$$\begin{aligned}&[C\ 0]\begin{bmatrix}(sI-A)^{-1} & -(sI-A)^{-1}K(sI-N)^{-1}\\ 0 & (sI-N)^{-1}\end{bmatrix}\\&=\frac{1}{\Delta}[C\ 0]\begin{bmatrix}\mathrm{adj}(sI-A)\det(sI-N) & -\mathrm{adj}(sI-A)K\,\mathrm{adj}(sI-N)\\ 0 & \mathrm{adj}(sI-N)\det(sI-A)\end{bmatrix}\\&=\frac{1}{\prod_{i=1}^{n}(s-\lambda_A(i))\prod_{j=1}^{r}(s-\lambda_N(j))}\\&\quad\times\left[C\ \mathrm{adj}(sI-A)\prod_{j=1}^{r}(s-\lambda_N(j))\right.\\&\quad\left.-C\ \mathrm{adj}(sI-A)\,D\ \mathrm{adj}(sI-N)\right]\end{aligned} \tag{22}$$

where $\Delta=\det(sI-A)\det(sI-N)$

In order that the system is completely observable, there should be no zero-pole cancellation. If a zero of $(s-\lambda_N(j))$ exists as a common factor in the transfer matrix, it must be also a common factor of $C\ \mathrm{adj}(sI-N)$.

The cancellation of a common factor $\mathrm{adj}(s-\lambda_N(j))$ means that it must be common factor of the polynomial matrix $[-C\ \mathrm{adj}(sI-A)\,K\,\mathrm{adj}(sI-N)]$. Therefore, from the above condition, the proof of Theorem 2 is completed. □

5. Simulation

Consider the following linear continuous system.

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 3\\ -2 & -1 & -3\end{bmatrix},\ C=[1\ 0\ 0]$$

For the above system, we choose the following two kinds of sinusoidal disturbance model arbitrarily to be added input of system as $d_1(t)=2\sin(6\pi t)$ and $d_2(t)=2\sin(6\pi t)+\cos(14\pi t)$.

For the disturbance models, $d_1(t)$, $d_2(t)$, the transfer function of sinusoidal disturbances is made by Laplace transformation. Consequently we obtain the system matrices of disturbance model, D_1 and $W_i(i=1, 2)$, from the transfer function. By using the obtained disturbance matrices, the gain matrices of general structured observer, K_i and $N_i(i=1, 2)$, are derived as

$$K_1=\begin{bmatrix} 0 & 37.6991 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\ N_1=\begin{bmatrix} 0 & -355.3058 \\ 1.0000 & 0 \end{bmatrix}$$

and

$$K_2=\begin{bmatrix} 0 & 37.6991 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ N_2=\begin{bmatrix} 0 & -355.3 & 0 & 0 \\ 1.000 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1934.4 \\ 0 & 0 & 1.000 & 0 \end{bmatrix}$$

Next, we design the general structured observer's gain H_i and $M_i(i=1, 2)$ by LQG design. The weighting matrices, Q_i and $R_i(i=1, 2)$, at quadratic performance criterion are chosen as

$Q_1=\text{diag}[0.001\ 0.001\ 0.001\ 0.001\ 0.001]$,
$R_1=1$,
$Q_2=\text{diag}[0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001]$, $R_2=1$.

Then, the observer's gain matrices are obtained as follows.

$$H_1=\begin{bmatrix} 0.0668 \\ 0.0006 \\ -0.0033 \end{bmatrix},\ M_1=\begin{bmatrix} -0.5969 \\ 0.0000 \end{bmatrix}$$

and

$$H_2=\begin{bmatrix} 0.1002 \\ 0.1002 \\ -0.0043 \end{bmatrix},\ M_2=\begin{bmatrix} -0.5969 \\ 0.0000 \\ 0.0035 \\ 0.0316 \end{bmatrix}$$

The eigenvalues of general structured observer, $\lambda_i(i=1, 2)$, for disturbance model 1 and 2 are given respectively as

$$\lambda_1=\{-2.7101,\ -0.1467\pm1.4810i,\ -0.0317\pm18.8496i\}$$

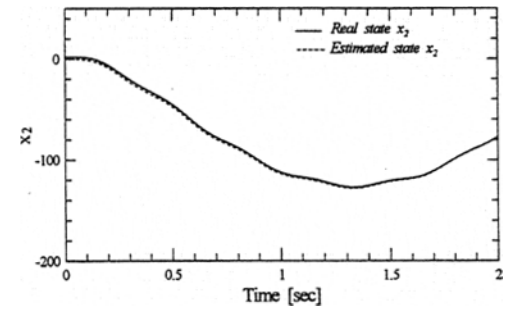

Fig. 2 Response of real and estimated state x_2 in case of disturbance model 1

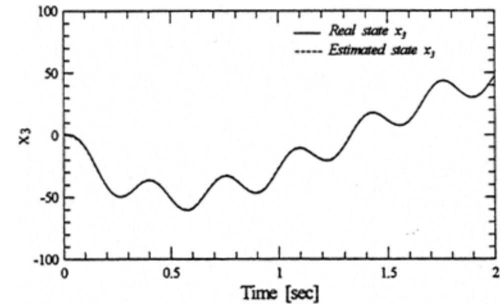

Fig. 3 Response of real and estimated state x_3 in case of disturbance model 1

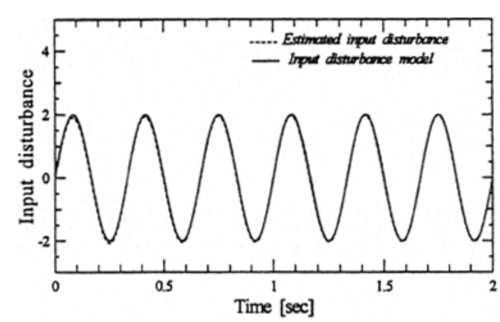

Fig. 4 Estimated result for disturbance model 1

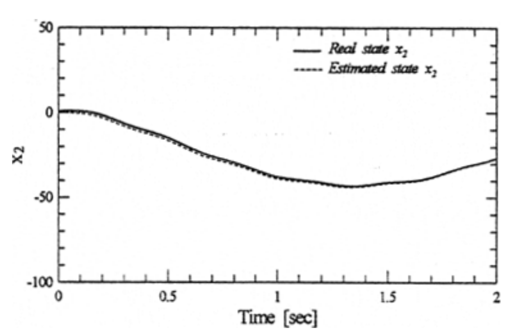

Fig. 5 Response of real and estimated state x_2 in case of disturbance model 2

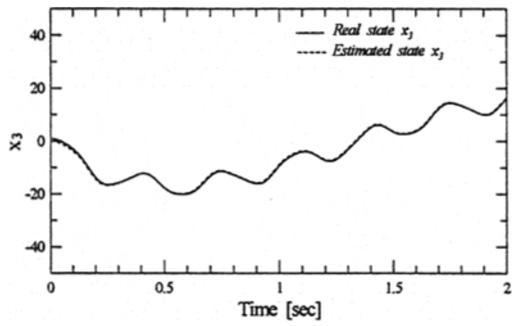

Fig. 6 Response of real and estimated state x_3 in case of disturbance model 2

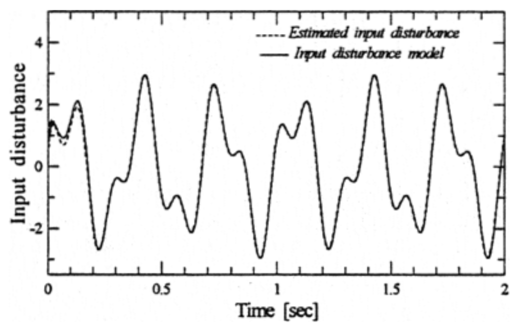

Fig. 7 Estimated result for disturbance model 2

and

$$\lambda_2 = \{-0.0158 \pm 43.9823i,\ -2.7102,\ -0.1475 \pm 1.4810i,\ -0.0317 \pm 18.8496i\}$$

On simulation, the sampling time is 1[ms]. The simulation results are shown in Figs. 2~7.

Figures 2~3 show that the estimated states $\hat{x}_2(t)$ and $\hat{x}_3(t)$ converge to real states for the disturbance model 1, and Fig. 4 represents the estimated disturbance. The estimated disturbance converge to the real disturbance effectively. Figures 5~6 show estimation results of states x_2, x_3 for the external disturbance model 2 which is more complicated and higher order than that of disturbance model 1. Also the estimated disturbance is given in Fig. 7. The simulation results indicate very effective characteristic of general structured observer design method.

6. Conclusion

In this paper, we have proposed general structured observer for linear time-invariant system. The proposed observer can estimate the states of given system in spite of external disturbances, and the disturbance can be estimated by using the estimated output error. Also, we have shown the existence condition of general structured observer's gain by the observability of system and the pole-zero cancellation of system's polynomial matrix. The simulation results show the effectiveness of the proposed observer for estimating the states of system and the external disturbance respectively.

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